

# A Method for Determination of Thermodynamic Dissociation Constants from Absorption Spectra with Applications to *ortho*- and *para*-Phenylenediamines\*

By Tosinobu ANNO and Akira SADÔ

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## Introduction

The dissociation constants of the mono-acidic bases or the mono-basic acids can be determined fairly well from the absorption curves of the substances in solution at different pH values<sup>1)</sup>, while with the dibasic acids and diacidic bases the spectroscopic determination of dissociation constants becomes more complicated<sup>2)</sup>. Thamer and Voigt<sup>2)</sup> have described a method for determining the dissociation constants in these cases. They measured the optical density of solutions of constant ionic strength at different pH values for a fixed wave length. The apparent dissociation constants are determined from these optical density data, and corrected to the true ones, using the Debye-Hückel equation.

We present here another method using the optical density data of solutions at only five different pH values. As an application of the present method, *ortho*- and *para*-phenylenediamines are chosen. There are a good many prior determinations of dissociation constants for the latter compound while the value of the second dissociation constant of the former compound is not found in the literature on the subject. The advantages and the disadvantages of the present method will also be discussed.

## Method

We shall describe the method only for the diacidic bases, the application to acids or amphoteric electrolytes being obvious. The following equation can be shown to hold<sup>2)</sup>:

$$D = \frac{Lc \left[ \epsilon_1 + \frac{k_{1b}}{a_{OH}} \epsilon_2 + \frac{k_{1b}k_{2b}}{a_{OH}^2} \epsilon_3 \right]}{1 + \frac{k_{1b}}{a_{OH}} + \frac{k_{1b}k_{2b}}{a_{OH}^2}} \quad (1)$$

where  $\epsilon_1$ ,  $\epsilon_2$ , and  $\epsilon_3$  represent the molar extinction coefficients of undissociated diacidic

base B, its singly and doubly dissociated ions  $BH^+$  and  $BH_2^{++}$ , respectively.  $D$  denotes the optical density of the solution of hydroxyl-ion activity  $a_{OH}$  and of analytical concentration  $c$  of the base in all its forms measured with an absorption cell of the length  $L$ .  $k_{1b}$  and  $k_{2b}$  are the apparent basic dissociation constants defined by

$$k_{1b} = \frac{a_{OH}[BH^+]}{[B]}, \quad (1a)$$

$$k_{2b} = \frac{a_{OH}[BH_2^{++}]}{[BH^+]},$$

where brackets indicate concentrations. According to Thamer and Voigt "If  $k_1$  and  $k_2$  overlap and  $k_1/k_2$  is therefore small, one can use optical density data and pH data with equation 4 (our equation 1) to obtain  $\epsilon_2$ ,  $k_1$ , and  $k_2$  by successive approximations. However, the calculations are very laborious". It is the purpose of this paper to show that using Eq. (1) the dissociation constants can be calculated *without* successive approximations. From Eq. (1) the following equation can easily be derived:

$$D'x - y + (D' - \epsilon_3)z/a_{OH} = (\epsilon_1 - D')a_{OH}, \quad (2)$$

where  $D' = D/Lc$ ,  $x = k_{1b}$ ,  $y = k_{1b}\epsilon_2$ , and  $z = k_{1b}k_{2b}$ .  $\epsilon_1$  and  $\epsilon_3$  are measurable as  $D'$  in alkaline ( $pOH \ll pk_{1b}$ ) and acidic ( $pOH \gg pk_{2b}$ ) solution, respectively. If the measurements are made of  $D'$  at three different medium pH values and constant ionic strength, Eq. (2) gives a set of linear simultaneous equations, in which  $x$ ,  $y$  and  $z$  are unknowns, for a fixed wave length. No iterative process is involved in solving these equations for  $x$ ,  $y$  and  $z$ , from which  $k_{1b}$ ,  $k_{2b}$ , and  $\epsilon_2$  can be determined. Although what values of pH are appropriate as "three different medium pH values" depends on the substance, the following general rule may be given:

If  $k_{1b}/k_{2b} \gg 1$ , (usually  $>10$ ), these pH values are chosen such that  $a_{OH} \cong \sqrt{k_{1b}k_{2b}}$ ,  $a_{OH} \cong k_{1b}$  and  $a_{OH} \cong k_{2b}$ .

If  $k_{1b}/k_{2b} \cong 1$ , they are all chosen such that  $pOH \cong pk_{1b}$  ( $\cong pk_{2b}$ ), usually with intervals of 0.3–0.5 pH units.

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1) For example see, Hughes, Jellinek, and Ambrose, *J. Phys. Colloid Chem.*, 53, 410 (1949).

2) B. J. Thamer and A. F. Voigt, *J. Phys. Chem.*, 56, 225 (1952).

### Experimental Results

The samples of *o*- and *p*-phenylenediamines used were commercial ones. Absorption curves were determined with a Beckman DU spectrophotometer with a fixed slit-width for each substance at a given wavelength. Values of pH were determined using a pH-meter supplied by Shimadzu Seisakusho with a Beckman glass electrode. They were converted into the hydroxyl-ion activity using the ionization constant of water  $K_w^{3)}$ . The optical density data are listed in Tables I and II for *o*- and *p*-phenylenediamines, respectively. They are limited to those corresponding to 10-90 percent transmittance. Since it is impossible to obtain the optical density data satisfying the above criterion with one concentration value of the solution, we must use two different concentration values for each substance and for each pH value. In acidic and alkaline extremes dilution by the solvent does not bring about the change of the hydrogen-ion concentration, since in such cases the buffer action of the solvent is strong. For intermediate pH values McIlvaine's<sup>4)</sup>  $\text{Na}_2\text{HPO}_4$ -citric acid buffer solutions with appropriate pH values were first prepared and then diluted to low ionic strength so that the Debye-Hückel equation can be applied. In this way, the buffer action was weakened and the pH value of the solution came to depend slightly upon the analytical concentration of the substance. Therefore, the optical density data of solutions with different concentrations are listed separately in the table. This supplied additional equations for the calculation of the dissociation constants and, as far as possible, two sets of equations were selected and used for calculation for a fixed wave length. The agreement between the values of the dissociation constants calculated from different sets of equations for a fixed wave length represents a measure of the experimental accuracy, particularly, of pH measurements. A preliminary investigation shows the dissociation constants of these substances to be: *o*-,  $k_{1b} \geq 10^{-10}$ ,  $k_{2b} = 10^{-14}$ ; *p*-,  $k_{1b} \geq 10^{-8}$ ,  $k_{2b} \geq 10^{-11}$ . From these figures appropriate values of pH are selected according to the criterion stated above. All measurements were made at room temperature. The control of the ionic strength was made with KCl. In the case of *o*-compound, where the relatively high ionic strength was necessary, dilution of the buffer solutions as prepared according to the indication in the literature was unnecessary. In this case the pH value does not depend on the concentration of the substance, since the buffer action of the solvent is strong. This is the reason why Table I for *o*-compound contains only one column for each intermediate pH value. The extinction coefficient  $\epsilon_2$  of singly charged ion  $\text{BH}^+$  can in principle be calculated, but the error included will be great compared with  $k_{1b}$  and  $k_{2b}$ , because, contrary to them,  $\epsilon_2$  cannot be determined as an average value, since  $\epsilon_2$  assumes, in

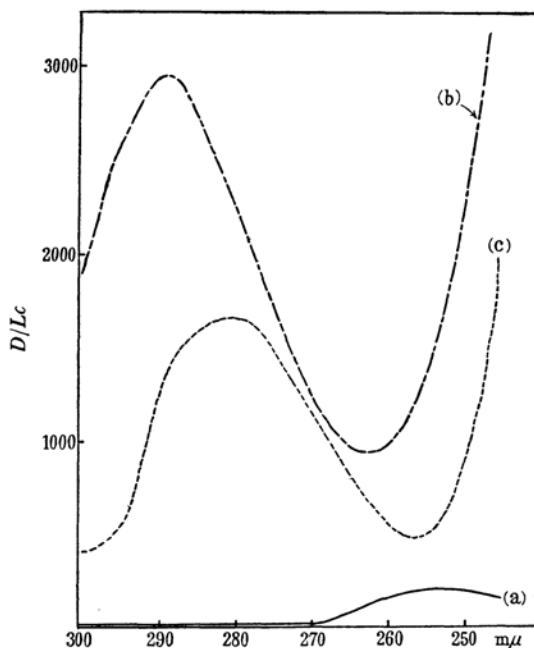


Fig. 1. Absorption curves of *o*-phenylenediamine in 6 N-HCl (a), 0.2 N- $\text{Na}_2\text{CO}_3$  (b), and pH 2.10 (c) solutions.

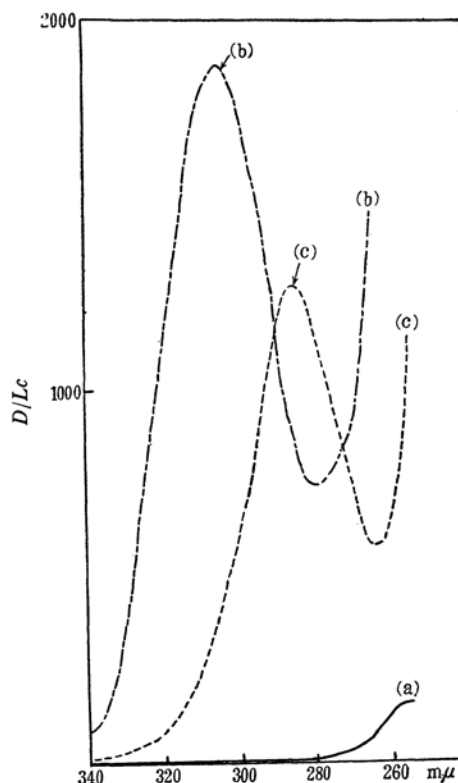


Fig. 2. Absorption curves of *p*-phenylenediamine in 1 N-HCl (a), 0.2 N- $\text{Na}_2\text{CO}_3$  (b), and pH 4.95 (c) solutions.

3) The values used for calculations are obtained by interpolation of the values reported by Harned and Hamer, *J. Am. Chem. Soc.*, 55, 2194 (1933).

4) T.C. McIlvaine, *J. Biol. Chem.*, 49, 183 (1921).

TABLE I  
THE VALUES OF  $D'$ , AND DISSOCIATION CONSTANTS OF *o*-PHENYLENEDIAMINE  
Temp.: 14°C,  $\mu$ : 1.0 mol./l.

Wave-length in m $\mu$	$D'$ Values					$k_{1b}$ $\times 10^{10}$	$k_{2b}$ $\times 10^{14}$
	6 N HCl	1.00 N HCl pH 0.00 <sup>a)</sup>	Na <sub>2</sub> HPO <sub>4</sub> -citric acid buffer solutions pH 2.10      pH 4.60		0.2 N Na <sub>2</sub> CO <sub>3</sub>		
290	9.5	125	1210	1640	2930	4.77	4.43
285	9.2	162	1550	1940	2800	4.11	4.21
280	12.0	175	1650	1940	2280	2.80	4.22
275	12.0	160	1490	1710	1690		5.44
270	16.0	131	1170	1340	1230		4.21
265	61.2	131	840	1000	970		4.78
260	130	166	560	710	950	2.84	5.20
255	172	192	475	700	1280	4.01	6.88
250	160	216	810	1200	2100	3.54	4.99
Mean						3.68 $\pm 0.32$	4.93 $\pm 0.30$

a) The value of the mean activity coefficient of 1.00 N-HCl is given in B. E. Conway, Electrochemical Data (Elsevier Publishing Company, Amsterdam, 1952), p. 75, to be 0.809 at 25°C which amounts to pH=0.10.

TABLE II  
THE VALUES OF  $D'$ , AND DISSOCIATION CONSTANTS OF *p*-PHENYLENEDIAMINE  
Temp.: 14.5°C,  $\mu$ : 0.049 mol./l.

Wave-length in m $\mu$	$D'$ Values								$k_{1b}$ $\times 10^8$	$k_{2b}$ $\times 10^{12}$	Columns <sup>a</sup> used for calc.	$k_{1b}$ $\times 10^8$	$k_{2b}$ $\times 10^{12}$	Columns <sup>a</sup> used for calc.
	1 N HCl	Na <sub>2</sub> HPO <sub>4</sub> -citric acid buffer solutions						0.2 N Na <sub>2</sub> CO <sub>3</sub>						
		pH 3.13	pH 3.00	pH 5.20	pH 4.95	pH 6.75	pH 6.70							
340	3	4		7		61	60	87	1.10		3, 5, 7	1.02		3, 5, 8
335	3	4		14		127	122	180	1.08		3, 5, 7	1.09		3, 5, 8
330	3.3	6		21		248	247	355	1.07		3, 5, 7	0.97		3, 5, 8
325	3.7	7		43		447	454	645	1.06		3, 5, 7	0.93		3, 5, 8
320	4.0	16		82		721	714	1034	1.12		3, 5, 7	1.03		3, 5, 8
315	4.3	34		144	109		1000	1432	1.02		3, 5, 8	1.12		3, 6, 8
310	4.7	71		250	213		1240	1760	1.07	7.56	3, 5, 8	1.08	8.19	3, 6, 8
305	5.0	130		407	375		1390	1880	1.02	8.67	3, 5, 8	1.02	9.38	3, 6, 8
300	5.3	217	170		606		1430	1820	0.99	10.05	3, 6, 8	0.95	10.86	4, 6, 8
295	5.6	335	280		906		1395	1590	0.89	10.50	3, 6, 8	0.87	9.60	4, 6, 8
290	6.1	437	378		1177		1292	1300		10.50	3, 6, 8		10.15	4, 6, 8
285	6.6	480	408		1283		1137	1015	1.46	10.68	3, 6, 8	1.47	10.13	4, 6, 8
280	7.4	455	388		1200		961	815	1.15	10.22	3, 6, 8	1.16	10.59	4, 6, 8
275	8.3	381	316		989		827	773	0.63	10.45	3, 6, 8	0.65	10.28	4, 6, 8
270	13.5	297	237		767		814	1035		10.37	3, 6, 8		10.93	4, 6, 8
265	53	256	199		592		1058	1600	2.49	11.34	3, 6, 8	2.57	12.01	4, 6, 8
260	114	297	243		628		1738	2620	1.73	9.87	3, 6, 8	1.73	12.37	4, 6, 8
255	150	490	443		1138		3120	4400	1.46	8.95	3, 6, 8	1.45	9.56	4, 6, 8
Mean									1.21 $\pm 0.11$	9.93 $\pm 0.30$		1.19 $\pm 0.11$	10.34 $\pm 0.33$	

a) The set of numbers in column 12 shows that for each wave length the set of  $D'$  values corresponding to the column indicated by these numbers is substituted in Eq. (2) and the values of  $k_{1b}$  and  $k_{2b}$  found in column 10 and 11, respectively, are calculated from the resulting set of three equations. Column 15 relates to columns 13 and 14 as column 12 does to columns 10 and 11.

general, different values at different wave lengths. For this reason, the calculated  $\epsilon_2$  values are not given in the tables. The observed  $D'$ -values of a solution whose pH value lies in the middle of  $pk_{1b}$  and  $pk_{2b}$  is a fairly good measure of the absorption by the species  $BH^+$  only, provided  $pk_{1b}$  and  $pk_{2b}$  are separated with each other by 3 or 4 as in phenylenediamines. The influence of the other species can be neglected at this value of pH. Figs. 1 and 2 show the absorption curves of *o*- and *p*-phenylenediamines, respectively, in acidic (a), alkaline (b), and medium pH (c) solutions. As was mentioned above, the curves (a), (b) and (c) are mainly due to species  $BH_2^{++}$ , B and  $BH^+$ , respectively. If at a given wave length the extinction coefficients of species B and  $BH^+$  are alike, the calculated value of  $k_{1b}$  at this wave length is probably in error. In the same way, if the extinction coefficients of species  $BH^+$  and  $BH_2^{++}$  are alike,  $k_{2b}$  calculated at this wave length is probably in error. Furthermore, the  $D'$  values corresponding to too high transmittance are likely to be in error. In fact, anomalous values are obtained from these data and these values are not given in the tables.

Thus the mean values of the apparent dissociation constants are:

$$o-, k_{1b} = (3.68 \pm 0.32) \times 10^{-10}, \quad k_{2b} = (4.93 \pm 0.30) \times 10^{-14};$$

$$p-, k_{1b} = (1.20 \pm 0.08) \times 10^{-8}, \quad k_{2b} = (1.01 \pm 0.03) \times 10^{-11}.$$

For the apparent dissociation constants obtained above, activity corrections were made, using the Debye-Hückel equation of the following form<sup>5)</sup>:

$$-\log \gamma_i = \frac{Az_i^2 \sqrt{\mu}}{1 + Ba_i \sqrt{\mu}},$$

$$A = \frac{1}{2.303} \frac{N^2 e^3}{(DRT)^{3/2}} \left( \frac{2\pi}{1000} \right)^{1/2},$$

$$B = \left( \frac{8\pi N^2 e^2}{1000 DRT} \right)^{1/2},$$

where  $e$  is the electronic charge,  $R$  is the gas

constants,  $D$  is the dielectric constant of water at the absolute temperature  $T$ ,  $N$  is the Avogadro's number, and  $\mu$  is the ionic strength of the solution<sup>6)</sup>.  $z_i$  and  $a_i$  are the charge and the radius of ion  $i$ .  $a_i$  is assumed to be 5 Å for both phenylenediamines in all their forms. The activity coefficients thus calculated for each experimental condition are collected in Table III.

TABLE III  
ACTIVITY COEFFICIENTS OF *o*- AND *p*-  
PHENYLENEDIAMINES CALCULATED BY  
DEBYE-HÜCKEL EQUATION

Isomers	<i>ortho</i>	<i>para</i>
Temp., °C	14	14.5
$\mu$ (mol./l)	1.0	0.049
$\gamma_B$	1.00	1.00
$\gamma_{BH^+}$	0.646	0.830
$\gamma_{BH_2^{++}}$	0.174	0.474

Then the final dissociation constants are calculated to be:

$$o-, K_{1b} = (2.4 \pm 0.3) \times 10^{-10},$$

$$K_{2b} = (1.3 \pm 0.1) \times 10^{-14} \text{ at } 14^\circ\text{C};$$

$$p-, K_{1b} = (1.0 \pm 0.1) \times 10^{-8},$$

$$K_{2b} = (5.8 \pm 0.1) \times 10^{-12} \text{ at } 14.5^\circ\text{C},$$

where the errors due to the Debye-Hückel equation are neglected. It is interesting to compare these results with the values obtained by other methods. They are listed in Table IV taken from Landolt-Börnstein Tables<sup>7)</sup>. It can be seen that the present method yields reasonable values of dissociation constants. Moreover, the present work adds the value of the second dissociation constants of *o*-phenylenediamine to the literature. Of course the precise value of  $K_{2b}$  of this compound is still uncertain since we were forced to make this determination at rather high ionic strength.

TABLE IV  
DISSOCIATION CONSTANTS OF PHENYLENEDIAMINES<sup>a</sup>

Isomers	Step	Temp., °C	Constant	Method	Author
<i>o</i> -	1st	21	$2.3_5 \times 10^{-10}$	Potentiometric	Kuhn & Wassermann Kuhn & Zumstein
<i>o</i> -	1st	25	$3.3 \times 10^{-10}$	Hydrolysis, partial extraction	Farmer & Warth
<i>p</i> -	1st	18	$1.3 \times 10^{-8}$	Conductmetric	Walden & Ulich
<i>p</i> -	1st	18	$1.1 \times 10^{-8}$	Potentiometric	Kolthoff & Bosch
<i>p</i> -	1st	ca. 20	$1.0 \times 10^{-9}$	Potentiometric & colorimetric	Pring
<i>p</i> -	1st	21	$9.5 \times 10^{-9}$	Potentiometric	Kuhn & Wassermann
<i>p</i> -	2nd	18	$3.5 \times 10^{-12}$	Potentiometric	Kolthoff & Bosch
<i>p</i> -	2nd	21	$1.5 \times 10^{-11}$	Potentiometric	Kuhn & Zumstein

a) Taken from ref. (7) in text.

5) For example see, D. A. McInnes, "The Principles of the Electrochemistry", Reinhold, New York (1939), Chap. 7.

6) The values of physical constants used in the calculation are taken from DuMond and Cohen, *Revs. Modern Phys.*, 25, 691 (1953), except for the dielectric

constant of water, for which the values are obtained by the interpolation of the values listed in S. Kaneko, *J. Electrochem. Soc. Japan*, 20, 273 (1952).

7) Landolt-Börnstein, "Physikalisch-Chemische Tabellen" Springer, Berlin (1931), 2te Ergänzungsband, p. 1094.

### Comparison with Thamer-Voigt's Method<sup>8,9)</sup>

As an application of the present method to the case where  $k_1$  and  $k_2$  are not separate from each other as for phenylenediamines, and also as a comparison of the present method with Thamer-Voigt's we have calculated the dissociation constants for isophthalic acid from the data presented by these authors<sup>2)</sup>. The results are given in Table V. The mean values of the dissociation constants calculated by both methods agree very well.

that the curve has a maximum or minimum. In the present method, the number of solutions to be prepared is considerably reduced. In addition, the above mentioned maximum or minimum are unnecessary. These are the advantages of the present method. While the error due to the optical density measurement becomes less by taking the mean over many values determined at different wave lengths, the error due to pH measurement cannot be reduced, which is the disadvantage of the present method.

In conclusion, the present method is experimentally easier to carry out especially

TABLE V  
OPTICAL DENSITIES<sup>a</sup> AND DISSOCIATION CONSTANTS OF ISOPHTHALIC ACID

Wave length in Å	Optical densities					$k_1 \times 10^4$	$k_2 \times 10^5$
	1.58	3.56	pH 4.18	5.095	8.72		
2476.0	0.987	1.134	1.152	1.036	0.982	3.21	3.62
2460.0	1.298	1.435	1.421	1.233	1.146	3.45	3.29
Wave length in Å	Optical densities					$k_1 \times 10^4$	$k_2 \times 10^5$
	1.58	3.37	pH 3.96	4.91	8.72		
2476.0	0.987	1.109	1.164	1.056	0.982	3.02	3.96
2460.0	1.298	1.410	1.446	1.256	1.146	2.71	4.52
Mean <sup>b</sup>						3.10	3.85
Mean (Thamer-Voigt) <sup>c</sup>						2.78	4.00

a) Optical density data are taken from ref. (2) in text.

b) Mean over four calculated values using the present method.

c) See ref. (2) in text.

In Thamer-Voigt's method it is true that the optical density measurements of many kinds of solution of different pH values reduce the errors due to optical density or pH measurements included in the final values of  $k_1$  and  $k_2$ , but the preparation of many kinds of solution is laborious especially for substances unstable in solution. Besides, it is necessary in their method that the optical density versus pH curve should show a decided maximum or minimum. In most cases a suitable wave length may be chosen so

for substances unstable in solution. For stable substances, the dissociation constants

9) After the preparation of this manuscript had been completed, a paper by B. J. Thamer (*J. Phys. Chem.*, 59, 450 (1955)) came to our attention. This method is essentially identical with ours excepting that  $\epsilon_1$  and  $\epsilon_3$  (in our notation) are not assumed to be the molar extinction coefficient at alkaline and acidic extremes. In diacidic bases (for example) having very small values of dissociation constants, his method may be particularly useful, since the direct determination of  $\epsilon_3$  necessitates the high value of ionic strength of the solution. If the optical density of the solution as acidic as possible (conforming to the requirement of low ionic strength) is practically equal to  $\epsilon_3$ , Thamer's method has no advantages because in his method a set of five equations containing five unknowns must be solved for a fixed wave length instead of a set of three equations in our method. If the optical density of the solution as acidic as possible is considerably different from  $\epsilon_3$ , his method may not give the precise value of the  $k_{2b}$ , because in such a solution rather a small fraction of the base exists in the form  $BH_2^{++}$  and even a small error in the optical density measurement results in a rather considerable error in  $k_{2b}$ . Therefore, for most practical purposes, our method involving the solution of a set of three equations for a fixed wave length may be sufficient.

8) It should be noted that  $k_1$  and  $k_2$  used in this section should be read as  $k_{1b}$  and  $k_{2b}$ , respectively, for diacidic bases such as phenylenediamines. For dibasic acids such as isophthalic acid  $k_1$  and  $k_2$  should be read as  $k_{1a}$  and  $k_{2a}$ , respectively, and defined by

$$k_{1a} = \frac{a_H[AH^-]}{[AH_2]}, \quad k_{2a} = \frac{a_H[A^{--}]}{[AH^-]}$$

where  $a_H$  is the H-ion activity and  $AH_2$ ,  $AH^-$  and  $A^{--}$  denote undissociated, singly and doubly dissociated dibasic acid, respectively.

may be determined by Thamer-Voigt's method a little more accurately.

The extension, of our method to polyelectrolytes appears possible, provided that each dissociated species has a different extinction coefficient in a proper wave-length region, although the number of simultaneous equations to be solved increases. For example, a tribasic acid requires a set of five equations.

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*Departments of Chemistry, Faculty of  
Science, Kyushu University,  
Fukuoka*

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